## PLASMA DRIFT-ANISOTROPY INSTABILITY AND ANOMALOUS TRANSPORT PROCESSES

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The author expounds a nonlinear theory of the instability of a weakly inhomogeneous plasma with hot ions when a "loss cone" is present in their velocity distribution.

Flute-type instabilities ( $k_Z\equiv 0$ ) are considered, which for a strong enough irregularity may build up even in short traps with "magnetic mirrors." It is shown that the total ion flux through the magnetic mirrors, which is caused by turbulent diffusion into the loss cone, exceeds by a factor of  $(2n/R_H \nabla n)^{1/2}$  the ion flux across the magnetic field as a result of diffusion in coordinate space (here n.  $\nabla n$ , are the density and its gradient,  $R_n$  is the ion Larmor radius). The diffusion time of ions into the "loss cone" is of the order  $\tau=10\Omega_n^{-1}(2n/R_n\nabla n)^{1/2}(\Omega_n$  is the ion Larmor frequency).

A plasma contained in magnetic traps is always in a nonequilibrium thermodynamic state. The nature of the nonequilibrium is connected with the specific geometry of the containing magnetic field. Here we will consider open traps with magnetic mirrors in which the nonequilibrium of the plasma is caused by:

- 1) the curvature of the magnetic field force lines and its associated effective gravity field;
- the localization of the plasma in a small volume, which brings about its inhomogeneity;
- 3) the presence of the so-called "loss cone" in the velocity distribution of the particles, in which the relation of the longitudinal and transverse velocities is such that they cannot be contained in the trap.

Under the influence of particle collisions the plasma tends to pass to a state of thermodynamic equilibrium. Here there is diffusion of the particles in velocity space and after virtually only one collision the particle falls into the "loss cone" and escapes from the trap.

The time of particle containment within the trap could be increased if collisions were made less frequent. However, in such a rarefied plasma various types of oscillation may arise spontaneously. The relaxation of the plasma state to one of thermodynamic equilibrium comes about much faster under the influence of these oscillations than relaxation due to collisions. Consideration of transport processes in a turbulent plasma enables us to estimate the real life time of particles in the trap, and this turns out to be less than would be expected from rare collisions.

Suitable choice of magnetic field geometry enables us to stifle the hydrodynamic instabilities associated with curvature of the force lines, and the weak kinetic instabilities arising from the excitation of low-frequency "drift wayes." Thus instabilities associated with the presence of a "loss cone" are more important.

Such an instability was first detected by Rosenbluth and Post [1]. Development of this instability leads to a state of affairs where under conditions close to optimal for thermonuclear reactions to take a normal course, anomalous diffusion of ions into the "loss cone" causes them to pass through the magnetic mirrors very rapidly [2]. However, the perturbations considered in [1] have the form of a wave  $\exp(-i\omega t + ik_z z)$  running along the magnetic field  $H_{0z}$ , and as shown in the same paper, perturbations of this type are strong damped in the region of the "magnetic mirrors," where the phase velocity of the perturbations becomes comparable with the thermal velocity of the electrons. This imposes a lower limit on the length of the systems in which instabilities of this type can develop  $L > L_c$  [1]. On the other hand, in short systems of length  $L < L_c$  flute-type oscillations exist  $(k_z \equiv 0)$  in the same region of frequency and wavelength, associated with density inhomogeneity [3]. Calculations similar to those in [1]

have shown that the presence of a "loss cone" also suffices for the development of these oscillations [4].

The critical radius  $R_C$  of the plasma volume, below which the instability develops, turns out to be, under conditions necessary for thermonuclear reactions to take place, of the same order as the critical length  $R_C \approx L_C \approx 10^2~R_H~(R_H~is$  the ion Larmor radius) [4]. Thus, in view of the requirement R < L it is impossible to obtain a stable plasma in which  $R > R_C$  and  $L < L_C$ . Instabilities of the latter type, which we will call drift-anisotropic instabilities, clearly limit the density of the plasma stably contained in existing devices [7].

Accordingly, \$1 gives equations describing the state of a weakly turbulent plasma of this type. In \$2 the energy distribution over turbulent pulsations with different scales is examined. The fluxes of ions through the "magnetic mirrors" and across the magnetic field are determined on the basis of the results of \$2 and \$3.

\$1. Basic equations. The description of a turbulent plasma may be taken to be complete if we are given the distribution of ions and electrons in coordinate and velocity phase space, and also the spectral distribution of energies resulting from the instability of oscillations in the space of the wave numbers  $(\mathbf{k}, \omega)$ . A correct mathematical description is possible in the case of weak instability ( $\gamma < \omega$ ,  $\gamma$  is the instability increment,  $\omega$  is the oscillation frequency). In this case we make use of quasi-linear equations for the ion and electron distribution functions and kinetic equations for the spectral density of oscillation energy, given for the general case of an inhomogeneous plasma in a magnetic field in the review [8]. These may be generalized to the case of a nonisotropic plasma with difficulty.

We confine ourselves to the case of a plasma layer with a density which varies in the direction of the x axis, situated in a strong homogeneous magnetic field

$$H_0 = \{0, 0, H_z\} \quad (H_0^2 \gg 8\pi n T_i)$$
.

Here n is the plasma density,  $T_i$  is the average energy of the ions. <sup>2</sup> We shall consider the electrons in the plasma to be cold  $(T_e \ll T_i)$ .

 $<sup>^1</sup>$ The paper in question does not consider the build up of flute-type instabilities  $k_z=0$ , which is possible in a homogeneous plasma either when sharp maxima are present in the velocity distribution of the ions [5], or when there is a considerable fraction of cold ions present f61.

 $<sup>^2</sup>$  The results of taking into account the effects of curvature, of the longitudinal and transverse irregularity of the magnetic field for oscillations close to harmonics of the cyclotron frequency  $\Omega_{\rm H}$ , and also an explanation of the part played by the cold plasma behind the mirrors may be found in [3]

Oscillations in the region of frequencies and wavelengths [1, 4]

$$\begin{split} &\Omega_{\rm H} \! \ll \! \omega \! \ll \! \omega_{\rm H}, \qquad k R_{\rm H} \! \gg \! 1 \! \gg \! k \! \rho_{\rm H} \; , \\ &\left(\Omega_{\rm H} = \! \frac{e H_0}{Mc} \; , \; \; R_{\rm H} \! = \! \frac{\sqrt{T_4 Mc}}{e H_0} \right), \quad \omega \, / \, k_z \! \gg \! \sqrt{T_e / m} \; , \quad (1.1) \end{split}$$

are considered.

Here  $\omega_n$ ,  $\rho_n$  are the Larmor frequency and ion and electron radius, respectively. Retaining the terms associated with the plasma inhomogeneity, we obtain for their equation in the linear approximation [8]

$$\begin{split} \varepsilon\left(\omega,\,\mathbf{k}\right) &\equiv 1 + \frac{\omega_{p}^{2}}{\omega_{n}^{2}} - \frac{\omega_{p}^{2}}{\omega^{2}} \frac{k_{z}^{2}}{k^{2}} - \frac{\Omega_{p}^{2}}{k^{2}v_{Ti}^{2}} \frac{\omega_{\star}}{\omega} + \\ &\quad + \frac{\Omega^{2}p}{k^{2}v_{Ti}^{2}} \left[\Psi\left(0\right) + F\left(\frac{\omega}{kv_{Ti}}\right)\right] = 0\,, \\ \Omega_{p}^{1} &= \left(\frac{4\pi e^{2}n}{M}\right)^{1/2}, \qquad \omega_{\star} &= \frac{k_{y}v_{Ti}^{2}}{\Omega_{n}} \frac{\nabla n^{2}}{n}\,, \\ F\left(y\right) &= 2\int_{0}^{\infty} \left(\frac{\partial \Psi}{\partial w} + \frac{\omega_{\star}}{2\omega}\Psi\right) \frac{dw}{\sqrt{1 - w/y^{2}}}\,, \\ w &= \frac{v_{\perp}^{2}}{n^{2}}, \quad v_{Ti} &= \left(\frac{T_{i}}{M}\right)^{1/2}. \end{split} \tag{1.3}$$

Here  $\omega_p$  are the ion and electron plasma frequencies, respectively,  $\omega_*$  is the drift frequency,  $v_{Ti}$  is the mean square ion velocity,  $\psi$  (w) is the distribution of ions over transverse velocities w normalized to unity. The required branch of the root for  $w > y^2$  is chosen according to [1]

$$\frac{1}{\sqrt{1-w/y^2}} \frac{iy_r}{\sqrt{w-y_r^2}}, \qquad y = y_r + i0.$$

The distribution of ion velocities is here assumed to be axially symmetric.

It follows from (1.2) that for an inhomogeneous plasma in addition to the term in the dispersion equation for oscillations which takes account of the electron inertia there is an extra term associated with the electron density inhomogeneity. The kinetic equation for the number density of oscillations in phase space  $n_k$  is also altered in a similar manner

$$n_{\mathbf{k}} \equiv \left| \frac{\partial \varepsilon \left( \omega, \, \mathbf{k} \right)}{\partial \omega} \right| \frac{k^2 \left| \, \varphi_{\mathbf{k}} \, \right|^2}{8\pi} \,. \tag{1.4}$$

Here  $\varphi_k$  is the amplitude of the k-th Fourier harmonic of the electric field potential of the oscillations. Just as in [2], decay processes play a basic part in the interaction of oscillations, while nonlinearity is again most significant in the electron motion equations [8]:

$$\begin{split} \frac{\partial n_{\mathbf{k}}}{\partial t} &= 2\gamma_{\mathbf{k}}n_{\mathbf{k}} + \sum_{\mathbf{k} = \mathbf{k}' + \mathbf{k}''} |V_{\mathbf{k}, \mathbf{k}', \mathbf{k}''}|^2 \left(n_{\mathbf{k}'}n_{\mathbf{k}''} - n_{\mathbf{k}'}n_{\mathbf{k}''}\right) \\ &- n_{\mathbf{k}}n_{\mathbf{k}'} \operatorname{sign} \frac{\partial \varepsilon_{\mathbf{k}''}}{\partial \omega_{\mathbf{k}''}} - n_{\mathbf{k}}n_{\mathbf{k}''} \operatorname{sign} \frac{\partial \varepsilon_{\mathbf{k}'}}{\partial \omega_{\mathbf{k}'}} \right) \delta \left(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} - \omega_{\mathbf{k}''}\right), \\ &|V_{\mathbf{k}, \mathbf{k}', \mathbf{k}''}|^2 = \frac{k^2}{8\pi} \left| -\frac{\omega_p^2 e \left[\mathbf{k}' \times \mathbf{k}''\right]_z}{k^2 m \omega_n \left(\omega' + \omega''\right)} \left\{ \left(\frac{k_y'' \nabla n}{\omega_n \omega'' n} - \frac{k_y' \nabla n}{\omega_n \omega' n}\right) + \\ &+ \left(\frac{k_z'}{\omega'} - \frac{k_z''}{\omega''}\right) \left(\frac{k_z}{\omega} + \frac{k_z'}{\omega''} \frac{k_z''}{\omega}\right) \right\} \left|^2 \left/ \frac{k'^2 k''^2}{(8\pi)^2} \right| \frac{\partial \varepsilon_{\mathbf{k}}}{\partial \omega_{\mathbf{k}'}} \frac{\partial \varepsilon_{\mathbf{k}'}}{\partial \omega_{\mathbf{k}'}} \frac{\partial \varepsilon_{\mathbf{k}''}}{\partial \omega_{\mathbf{k}'}} \right| (1.5) \end{split}$$

Here a small term describing the resonance absorption by ions of two oscillations at once [2] is omitted for the sake of simplicity.

Qualitatively new effects appear when the relaxation of the plasma particle distribution under the influence of oscillations is examined. This is connected with the fact that in addition to the anisotropy of ion velocities the plasma inhomogeneity also becomes a significant factor in the development of instabilities. Thus the development of instability should lead not only to the diffusion of ions in velocity space but also to their diffusion across the magnetic field in coordinate space. The quasi-linear equations for ions and electrons have the form [8]

$$\begin{split} \frac{\partial \psi \left( x, \, v_{\perp} \right)}{\partial t} &= \frac{e^{2}}{M^{2}} \sum_{\mathbf{k}} \left( \frac{\partial}{v_{\perp} \partial v_{\perp}} + \frac{k_{y}}{\omega_{\mathbf{k}} \Omega_{n}} \frac{\partial}{\partial x} \right)_{k_{\perp}} \frac{\omega_{\mathbf{k}}^{2} \left| \varphi_{\mathbf{k}} \right|^{2}}{\sqrt{v_{\perp}^{2} - \omega_{\mathbf{k}}^{2} / k^{2}}} \times \\ &\times \left( \frac{\partial}{v_{\perp} \partial v_{\perp}} + \frac{k_{y}}{\omega_{\mathbf{k}} \Omega_{n}} \frac{\partial}{\partial x} \right) \psi \left( x, \, v_{\perp} \right) \frac{\partial f_{0v}}{\partial t} = \\ &= \frac{e^{2}}{m^{2}} \sum_{\mathbf{k}} \left( k_{z} \frac{\partial}{\partial v_{z}} - \frac{k_{y}}{\omega_{n}} \frac{\partial}{\partial x} \right) \frac{\gamma_{\mathbf{k}} \left| \varphi_{\mathbf{k}} \right|^{2}}{(\omega_{\mathbf{k}} - k_{z} v_{z})^{2} + \gamma_{\mathbf{k}}^{2}} \left( k_{z} \frac{\partial}{\partial v_{z}} - \frac{k_{y}}{\omega_{n}} \frac{\partial}{\partial x} \right) f_{0e}. \end{split}$$

$$(1.6)$$

Here as in [2] the rotation of the ions in the magnetic field has been averaged over angle in the equation for ions, and an average has also been taken of their longitudinal velocities  $v_{\rm Z}$ ; moreover, only the resonance interaction of ions with oscillations has been taken into account. The electrons have only nonresonant interactions with oscillations.

§2. Turbulence spectrum. The turbulence spectrum in a homogeneous plasma for oscillations with  $k_Z\neq 0$  was found in [2]. Here we shall dwell on the case of flute-type oscillations  $k_Z\equiv 0$ . Since in view of the validity of the dispersion relation (1.2) both the ratio  $k_Z/k$  and the ratio  $\omega*/\omega$ , in the case under consideration, may be expressed in terms of the value of the oscillation wave number  $k_\perp$ , neither of these ratios need be introduced. In this case the spectral density of energy depends [2] only on the wave vector  $k_\perp$  across  $H_0$  and the phase velocity  $y_k\equiv \omega_k/k_\perp v_{Ti}$ ,

$$\sum_{\mathbf{k}} \frac{e^2 | \varphi_{\mathbf{k}} |^2}{M^2 v_{T_4^4}} \approx 0.1 \frac{\gamma_{\mathbf{k}} y_{\mathbf{k}}^2}{\omega_{\mathbf{k}} \mathbf{k}_{1}^2 R_{1i}^2} \qquad \left( y_{\mathbf{k}} = \frac{\omega_{\mathbf{k}}}{k_{\perp} v_{T_4}} \right). \quad (2.1)$$

In a weakly inhomogeneous plasma the phase velocity determined by the dispersion relation (1.2) is very small  $y_k \ll y_m$  ( $y_m$  is the value of the arguments for which the function F(y) has a positive maximum). Thus for F(y) we may employ the approximate formula

$$F(y) \approx iy$$
s,  $\sigma \equiv 2 \int_{0}^{\infty} dw \left( \frac{d\psi}{dw} + \frac{\omega_{*}}{2\omega} \psi \right) w^{-1/2}$ . (2.2)

Oscillations with  $k_z \equiv 0$  have the frequency [2]

$$\omega_{\mathbf{k}} = \frac{\omega_{*}}{k^{2}\lambda_{D}^{2} + \psi(0)} \left[ 1 + \frac{iy_{\mathbf{k}} \omega_{\mathbf{k}} \sigma}{\omega_{*}} \right],$$

$$\lambda_{D} = \frac{v_{Ti}}{\Omega_{p}} \left( 1 + \frac{\omega_{p}^{2}}{\omega_{n}^{2}} \right)^{1/2}.$$
(2.3)

Here  $\lambda_D$  is the Debye radius. Whence it is clear that oscillations increase for  $\sigma>0$ . This occurs at least for an empty loss cone, when  $\psi$  (0) = 0.  $\sigma\sim 1$  for  $\omega\geqslant\omega_A$  [4]. The increment increases as the wavelength increases and reaches its maximum value

$$\gamma_{\max} \approx \omega \approx \Omega_{p_*} \left(\frac{R_n \nabla n}{2n}\right)^{3/4} \left(\frac{2}{\sigma}\right)^{1/4}, \quad \Omega_{p_*} = \frac{\Omega_p}{1 + \omega_p^{-2} / \omega_n^{-2}} \quad (2.4)$$

for wavelengths

$$k_m \lambda_D \approx \left(\frac{R_n \nabla n}{n} \, \sigma\right)^{1/4}$$
 (2.5)

In the shorter wavelength region  $\lambda_D^{-1} > k > k_m$  the increment decreases as  $\gamma = \gamma_{max} (k_m/k)^5$ . In the longwave part of the spectrum the two terms on the right hand side of equation (2.3) will be important, so that the increment again falls

$$\omega \equiv \omega_r + i\gamma = (1+i) k v_{Ti} \left( \frac{R_n \nabla n}{2\sigma n} \right)^{1/2}, \quad k \leq k_m. \quad (2.6)$$

The condition  $\gamma_{max} \geqslant \Omega_n$  determines the critical density above which instability develops [4]<sup>1</sup>:

$$\frac{R_{\nu} \nabla n}{n} \geqslant 2 \left[ \frac{\Omega_{\nu}^{2}}{\Omega_{\nu}^{2}} + \frac{m}{M} \right]^{3/3} \left( \frac{\sigma}{2} \right)^{1/3}. \tag{2.7}$$

We note that for still larger gradients  $n^{-1}R\nabla n>$   $>(\Omega_n / \Omega_{p_s})^{1/s}$ , i. e., oscillations with wavelength  $k\lambda_D\sim$   $\sim$  1 develop even in a Maxwellian plasma.

Over a considerable region of wave numbers  $k < < k_m$  the instability is strong ( $\gamma \sim \omega$ ), and consequently it is impossible to establish strictly equations (1.5) and (1.6) obtained for a weakly turbulent plasma. We shall thus consider an idealized situation when  $\psi$  (0)  $> k_m^2 \lambda_D^2$ , and consequently  $\gamma < \omega$ . The results for the case of an "empty loss cone"  $\psi$  (0) = 0 may be obtained by going to the limit  $\psi$  (0)  $\rightarrow$  0 in the solutions of equations (1.5), (1.6).

It follows from (2.1), (2.3), (2.6) that the spectral density of energy in the region of short-wave pulsations decreases very rapidly:

$$\sum_{\mathbf{k}} |\varphi_{\mathbf{k}}|^2 \sim \frac{1}{k^{10}} \qquad (\lambda^1 > k > k_m). \qquad (2.8.1)$$

In the region of long-wave pulsations it increases, but very slowly:

$$\sum_{k} |\varphi_{k}|^{2} \sim \frac{1}{k^{2}} \qquad (k < k_{m}) . \qquad (2.8.2)$$

Very long wavelength oscillations for which the growth increment becomes equal to the ion gyrofrequency  $\gamma_k < \Omega_n$ , cannot be considered on the basis of (1.2). This occurs for wavelengths greater than

$$k < k_c = \frac{1}{P_n} \left( \frac{2\sigma n}{R_n \nabla n} \right)^{1/2} . \tag{2.9}$$

\$3. Anomalous escape of ions from the trap. We shall now consider transport processes leading to the escape of particles from the trap. These are, first of all, the diffusion of ions in velocity space, when ions of low energy at the border of the "loss cone" give up part of their energy to the oscillations and fall into the "loss cone", subsequently escaping from the traps through the magnetic mirrors. The energy of oscillations in the stationary mode is damped by the ions with large transverse energy and leads to their acceleration.

The diffusion coefficient for the main portion of the ions with velocities  $v_{\perp} \sim v_{Ti}$  is given by the same expression as in [2]

$$D_w \approx \sum_{\mathbf{k}} y_{\mathbf{k}} \omega_{\mathbf{k}} \frac{e^2 |\varphi_{\mathbf{k}}|^2}{M^2 v_{T_i}^4} \approx 0.1 \frac{\gamma_{\mathbf{k}} y_{\mathbf{k}}^3}{k^2 R_n^2} , \qquad (3.1)$$

where  $\gamma_k$ ,  $y_k$  and the minimal scale of turbulence is determined by (2.6), (2.9). In the limit of an empty "loss cone" we obtain for the time for the plasma to escape from the trap  $\tau \approx D_w^{-1}$ 

$$\Omega_n \tau = 10 \left( \frac{2 \operatorname{sn}}{R_n \nabla n} \right)^{\mathrm{s}/2}. \tag{3.2}$$

Integrating the quasi-linear equations (1.6) with respect to velocities, we arrive at the conclusion that in addition to the plasma flux through the magnetic mirrors a strong diffusion of ions across the magnetic field may be observed. The ratio of the total particle flux across the magnetic field  $j_\perp$  and along it  $j_\parallel$  turns out to be independent of the level of turbulent pulsations

$$\frac{i_{\perp}}{i_{\parallel}} \equiv \frac{\omega_{*}}{2\omega} \Big|_{k=k_{c}} \sim \left(\frac{\sigma R_{n} \nabla n}{2n}\right)^{1/2} \ll 1.$$
 (3.3)

It is clear from this relation that in [3] the diffusion of ions across the magnetic field was wrongly regarded as the chief process leading to the escape of particles from the trap. We note that for the case when perturbations with  $k_z \not\equiv 0$  may develop, this ratio decreases still further:

$$\frac{i_{\perp}}{i_{\parallel}} \approx \left(\frac{\omega_*}{\omega}\right)^2 \sim \left(\frac{R_n \nabla n}{y_m^n}\right)^2$$
 (3.4)

Diffusion into the "loss cone" proceeds very strongly, so that for trap lengths greater than

$$L > L_* = 10 \left( \frac{25n}{R_n \nabla n} \right)^{6/2} R_n$$
 (3.5)

the "loss cone" fills up and the time of escape increases to the transit time of thermal ions between the magnetic mirrors

$$T \sim L / v_{Ti}$$
. (3.6)

This result is valid in those cases when slow ions  ${\rm v_Z} \ll {\rm v_{Ti}}$  cannot accumulate in the system (for example, due to the presence of a positive space charge

This criterion was obtained in [3] for a monoenergetic ion velocity distribution  $\psi = \delta(w - w_{(0)})$ .

of ions arising as a result of the rapid escape of electrons during electron-electron collisions).

Otherwise there are never any slow ions that do not manage to escape from the trap during the diffusion time into the "loss cone" ( $v_Z \leqslant L/\tau$ ). Their accumulation in the "loss cone" may lead to a considerable decrease in the rate of decay of the plasma, if the number of particles with an increase of  $v_Z$  does not fall off exponentially (in this case the longitudinal energy  $\langle 1/2\,\mathrm{M}v_Z^{\ 2}\rangle$  turns out to be much less than the transverse  $\langle 1/2\,\mathrm{M}v_L^{\ 2}\rangle$  and this may lead to the development of strong hydrodynamic instabilities).

The stability of a plasma with a large number of slow ions when  $\varphi$  (0) ~ 1, with respect to the build-up of oscillations with  $k_z \neq 0$  has been treated in [9]. The generalization to the case of flute-type instabilities  $k_{\rm Z} \approx 0$  is trivial. Without dwelling in detail on the particular case of such a plasma, we will consider some results. As in [9], the distribution with  $\psi(0) \neq 0$ is stable even when a loss cone is present and becomes unstable only when allowance is made for the rare ion-ion collisions which lead to a Maxwell distribution of longitudinal ion velocities in the small region  $v_z < \Delta v$ . In this case only oscillations with a phase velocity  $y_k < y_m \sim \Delta v/v_{Ti}$  develop. If  $y_m \geqslant$  $\gtrsim \sqrt{R_n \nabla n / n}$ , then no change in the theory need be introduced. Otherwise the instability becomes weak as follows from (2.3).

The maximal increment is attained for  $k\lambda_D \gg (R_n \nabla n / y_m n)^{1/2}$ :

$$\begin{aligned} \omega + i \gamma &\approx \Omega_{p*} \frac{R_{n} \nabla n}{k \lambda_{D} n} \left( 1 + i \frac{y_{\mathbf{k}^{2}} n \sigma}{R_{n} \nabla n} \right) \lesssim \\ &\lesssim \left( \frac{y_{m} R_{n} \nabla n}{n} \right)^{1/2} \Omega_{p*} + i y_{m}^{5/2} \left( \frac{R}{R_{n} \nabla n} \right)^{1/2} \sigma \Omega_{p*} \end{aligned} \tag{3.7}$$

and falls off as  $y_k$  decreases. Comparing the diffusion time of ions into the loss cone  $^{\text{1}}$ 

$$\tau \approx \tau_D y_m^{-13/2}, \qquad \tau_D \approx \frac{10\Omega_{p*} (R_n \nabla n / n)^{3/2}}{\Omega_n^2 \sigma}$$
 (3.8)

with the relaxation time of the distribution in the region  $v_{\mathbf{Z}} < y_m \, v_{Ti}$ 

$$aupprox au_{i/i} y_m^2, \qquad au_{i/i}pprox rac{M^{1/2}T_i^{9/2}}{ne^4\lambda}, \qquad \lambda\sim 20$$
 ,

we find the region in which the distribution function is Maxwellian

$$y_m \approx \left(\frac{\tau_D}{\tau_{i/i}}\right)^{2/in}.$$
 (3.9)

$$w < R_{_H} \nabla n T_i/n$$
.

In this case the containment time is  $\tau \approx \tau_{i/i}^{13/17} \tau_D^{4/17}$ . The approximation of an empty "loss cone" is not violated since in short traps L <  $y_m v_{Ti} \tau$ . It follows from the expression for the increment and from the last formula that for very high temperatures the instability increment in a plasma with a large number of slow ions becomes comparable with  $\Omega_{\rm R}$ , and as the temperature increases further the instability will not develop.

Summing up the results of the paper we may draw attention to the following characteristics of the containment of a dense plasma with hot ions in a mirror machine.

- 1. If the plasma density exceeds the critical, then the development of an instability leads to the anomalously fast escape of the ions through the "magnetic mirrors" in the time (3.2). When this occurs the density falls to the critical value (2.7).
- 2. The ion flux across the magnetic field is less then the corresponding flux through the magnetic mirrors.
- 3. The transverse energy of the ions remaining in the trap increases with time on account of the decrease of the transverse ion energy of those ions escaping through the "magnetic mirrors."

To obtain a clearer idea of the size of the real limit in density  $n \leqslant n_{0\,\text{C}}$  which may be attained in existing traps with magnetic mirrors, we give an estimate of  $n_{0\,\text{C}}$  for two devices IP-5 [7] and "O  $\Gamma$ PA-II" [10].

For typical conditions of the apparatus IIP-5 ( $\rm H_0 \approx 4000~Oe$ ,  $\rm R_H \ \nabla \ n/n \approx 0.25$ ,  $\rm T_i \approx 5~keV$ , max  $\rm n \sim 10^{11}$ ,  $\rm L = 120~cm$ ) the length of the device is less than the critical and the value of the critical density  $\rm n_{0C}$  is found from (2.7). The quantity  $\rm n_{0C} \approx 10^{10}~cm^{-3}$  has limited the density of plasma contained for a time in the trap of recent experiments [7], regardless of the fairly high density of the injected plasma. The decay time of a plasma with density  $\rm n \sim 10^{11}~cm^{-3}$  coincides in order of magnitude with (3.2). All this tends to suggest that drift-anisotropy instability [7] occurred in experiments [7]. The Mikhailovskii-Timofeev criterion of drift-cyclotron instability [3] gives a higher limit for the density; the diffusion time for particles across  $\rm H_0$  is in this case greater than (3.2). Thus its manifestation in the experiments is apparently weaker.

For typical parameters of "OFPA-II" ( $T_i$  = 75 keV, L = 200 cm, R = 28 cm,  $H_0 \approx 1.5 \cdot 10^4$  Oe) the length of the apparatus also turns out to be less than the critical. The critical density  $n_{0C} \approx 10^{12}$  cm<sup>-3</sup> has not yet been attained in this device.

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<sup>&</sup>lt;sup>1</sup>The same expression must be applied for ion distributions in which the decrease of the number of ions with transverse energy starts from very small energies only

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